

# **Modelling Church Growth: A Systems Approach**

## **Part 2**

**The Unlimited Enthusiasm Church Growth Model  
with Births, Deaths and Reversion**

**Technical Report UG-M-00-2**

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## Abstract

*The growth of a church is investigated using the notation of systems dynamics. It is hoped that this approach will be more accessible than a strict mathematical one.*

*Part 1: The systems dynamics method is explained from scratch (chapter 2). This is then applied to a simple model of church growth where the population is split between unbelievers and believers who are all enthusiasts involved in the conversion process. Conversion is driven by contact between enthusiasts and unbelievers (chapter 3). These enthusiasts are assumed to retain their conversion potential throughout their lifetime. As such the whole population gets converted to the church.*

*Part 2: The model is made more realistic by including births and deaths (chapter 4), and reversion (chapter 5). If the church fails to recruit all its own children, or if it loses people after conversion, then the whole population does not get converted. As such the church numbers settle at a value less than the total population depending on the strength of these effects. If the effects are extreme the church becomes extinct.*

*Part 3: The model is modified so that enthusiasts lose their enthusiasm, i.e. their conversion potential, after a length of time (chapter 6). The church is now split into enthusiasts and inactive believers. Again the whole of the population fails to be converted. Including birth death and reversion effects into this model compound these problems (chapter 7). The model is applied to the current state of the Christian Church and some past revivals*

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## 4 Births and Deaths

### 4.1 Deaths

The church growth model given in chapter 3<sup>1</sup> will not hold for long time periods as both believers and unbelievers will die and be born. The standard model for deaths is given by the draining process (section 2.2.2) - a balancing loop that decays to 0 (figure. 2.3g). That is the negative exponential model. It is assumed that both believers and unbelievers die at the same rate. Thus there is no persecution causing the early death of believers, nor are there any long life advantages in being a believer!

*Assumption 7.1a*      *Believers and Unbelievers die at the same rate*

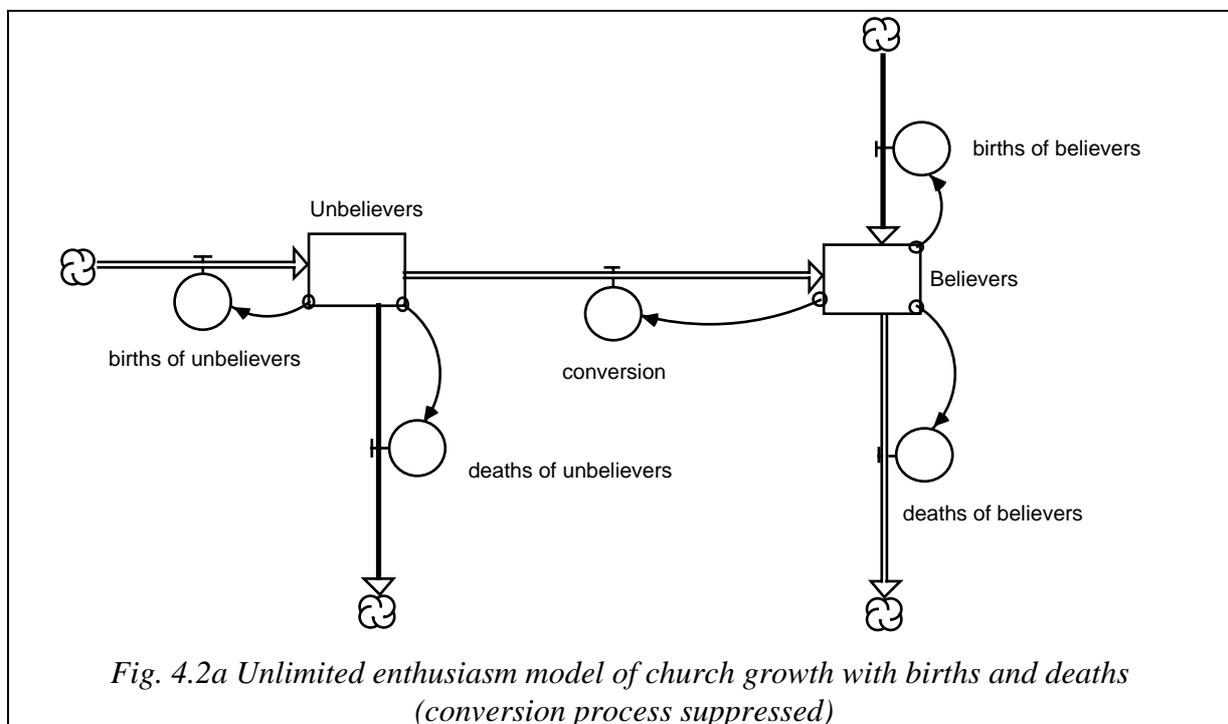
This introduces a new parameter into the model -the death rate.

### 4.2 Births

The standard model for births is the compounding process (section 2.2.1) - a reinforcing loop (fig. 2.3c). This is the exponential model. Again it is assumed that both believers and unbelievers have the same birth rate. Thus any birth control practices of the religious group are the same as the population at large.

*Assumption 7.1b*      *Believers and Unbelievers are born at the same rate.*

Thus the conversion model of figure 3.2e is extended to include input, representing birth, and output, representing death, to both stocks:



<sup>1</sup> Chapters 1-3 are given the technical report UG-M-00-1.

Now births produce an addition to the population. Are these additions the same population as their parents? That is, are the children of believers automatically believers, and those of unbelievers automatically unbelievers?

Clearly a child is not born with a religious belief. However if a child is brought up in the faith and subsequently adopts that faith while still in childhood, or adolescence, then they will be regarded as being born as believers. This is often called biological growth to distinguish it from conversion growth. Such children are not converted into the church having been an unbeliever.

Recognising the fact that not all the children of believers will become believers then a parameter representing the fraction of children of believers who are born believers is introduced.

*Assumption 7.2*                      *Children of believers may be born as believers or unbelievers*

However the same is not normally true for the children of unbelievers. Thus:

*Assumption 7.3*                      *Children of unbelievers are always born unbelievers.*

There are circumstances where this may not always be true e.g. in societies where unbelievers are willing to send children to Sunday School or Christian day schools. However these mechanisms are disregarded here.

Thus two more parameters are introduced into the model - the birth rate and the fraction born believers.

For believers and unbelievers the number of people born will depend on both these parameters. However if the fraction born believers is 0 then there are no new believers apart from conversion. Thus

$$\text{births\_of\_believers} = \text{fraction\_born\_believers} * \text{birth\_rate} * \text{Believers}$$

The detailed input and output for believers and unbelievers which encapsulate assumptions 7.1-7.3 are given in figures 4.2b and 4.2c.

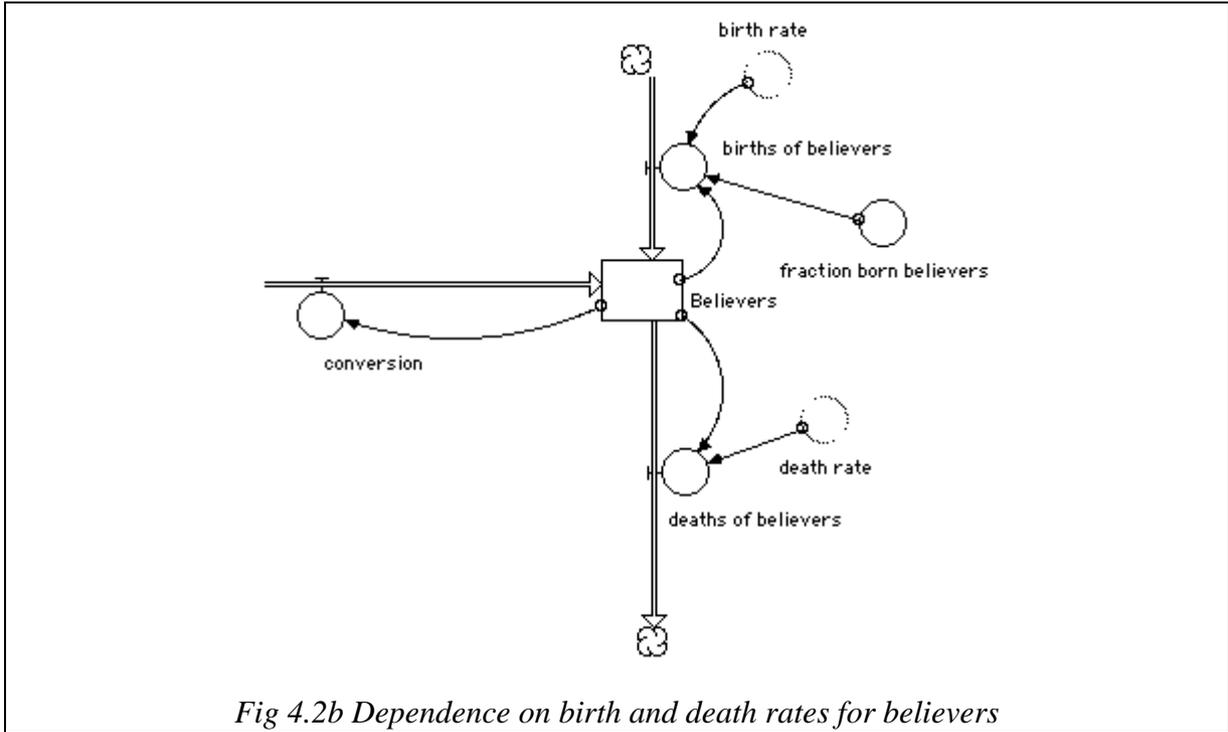


Fig 4.2b Dependence on birth and death rates for believers

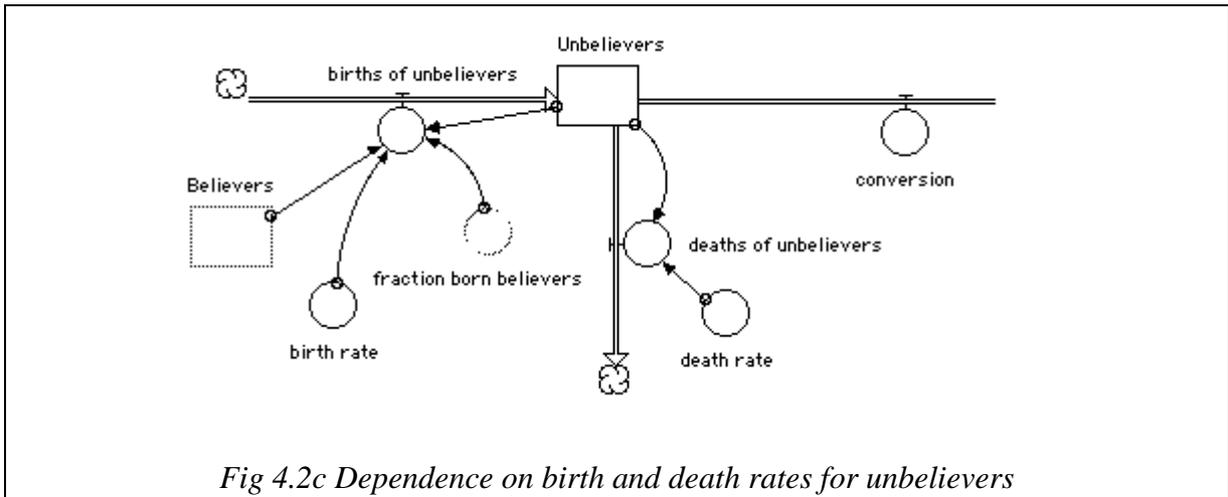
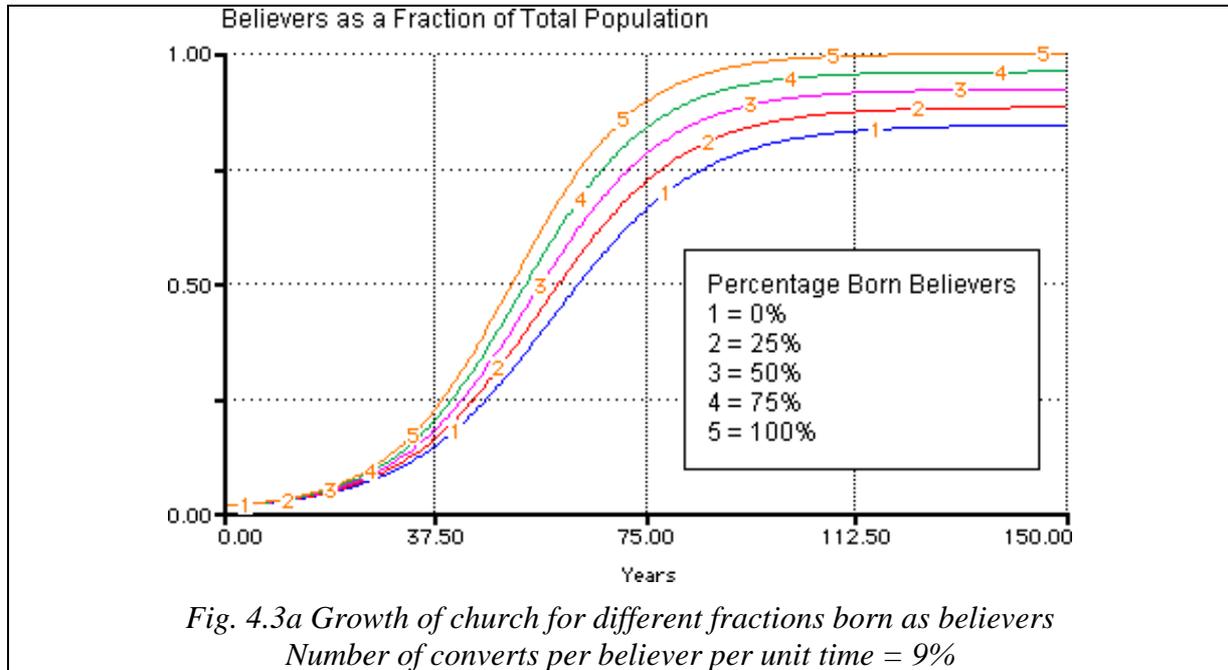


Fig 4.2c Dependence on birth and death rates for unbelievers

## 4.3 Analysis

### 4.3.1 Equal Birth and Death Rates

In the first instance it will be assumed that the birth and death rates are equal, so that there is no change in the total population. The solution can be investigated for different fractions born believers, as in figure 4.3a. The solution is still S-shaped.

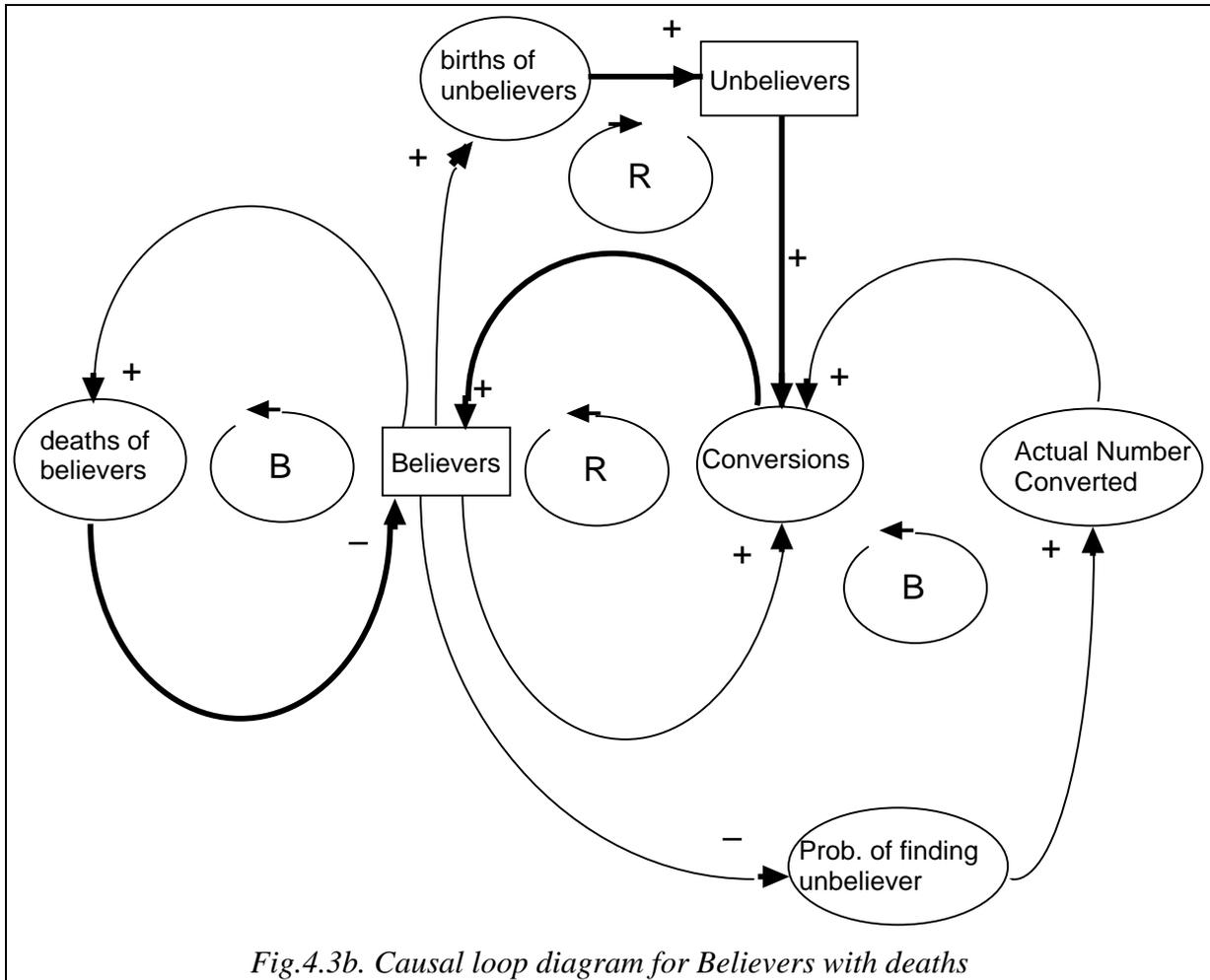


Whatever the fraction of children kept by the church, the population of believers increases to a stable fixed value. However the whole population is no longer converted unless all the children of believers become believers themselves. Anything less and the church can only grow to a value less than 100% of the whole population, a value which is independent of the initial percentage of believers. Thus the inability of the church to hold its own children in the faith will prevent it from converting the whole population over time. The worst case occurs if the church has to rely on conversion growth alone.

This limit to the growth of the church can be seen from the causal loop diagram figure 4.3b. Consider the worst case where none of the children of believers are born believers (curve 1 in figure 4.3a). The believers are now subject to a second balancing loop through its deaths. For growth to stop all its rates must balance out. Thus “conversions” will no longer be zero, but will be an amount needed to balance the deaths. Thus the probability of finding a believer on which conversions depends cannot be zero either so there must be some believers left. This is sometimes referred to as the “drifting goal” archetype. The “goal” for the believers is the whole population. However the death process will cause this goal to drift to a lower value. Only if all the births are straight into the believers category, exactly cancelling their deaths, will the effect be wiped out and goal be the total population (curve 5 in figure 4.3a).

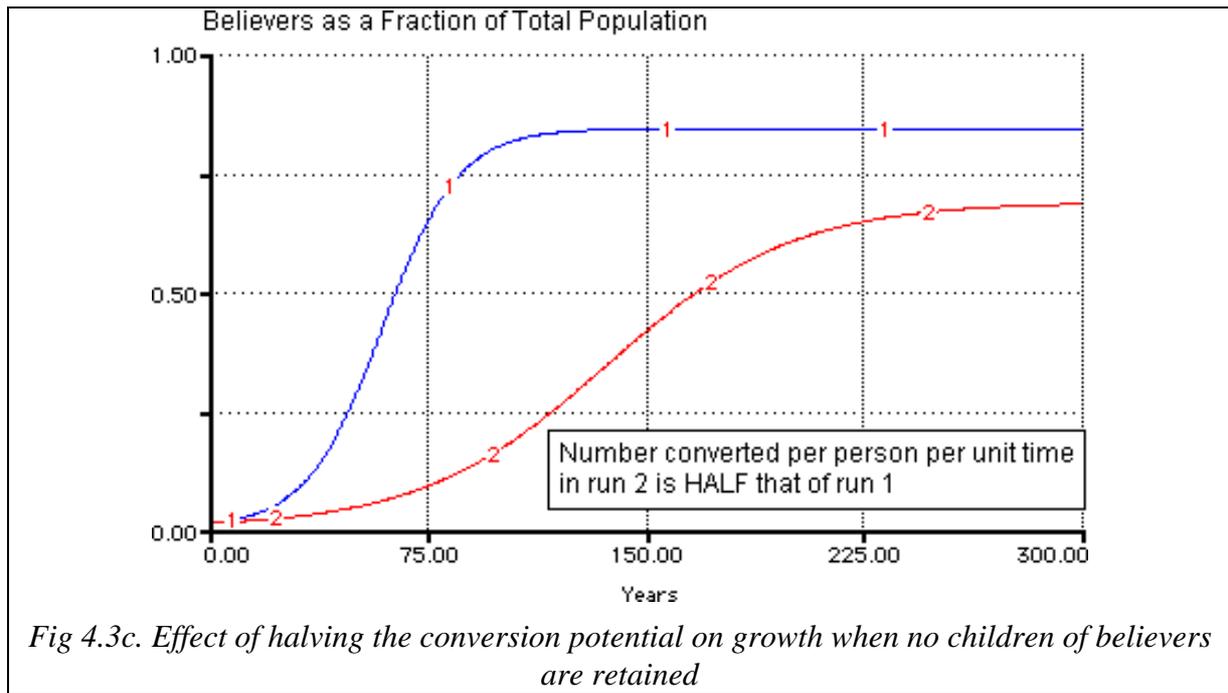
Of course both conversion and the death rate being zero would also balance the system. In this case the unbelievers would have become extinct. However there is an additional

reinforcing loop through births into the unbelievers equal to the deaths which ensures that the conversions cannot become zero<sup>2</sup>. The pool of unbelievers is being replenished at the same rate that believers are losing. Thus the drifting goal is non-zero



If the church cannot hold all its own children then, the lower the conversion potential the lower the stable percentage the church reaches in society, and the longer it takes to do so. Halving the number converted per believer per unit time results in a drastically slower growth. It also sees the believers' final share of the population drop from 84% to 68% for the case where no children are retained by the church. See figure 4.3c.

<sup>2</sup> There is another reinforcing loop through births to unbelievers and then via the probability to conversions. This is not given in figure 4.3b. Its effect on the dynamics is the same as the main loop through unbelievers.

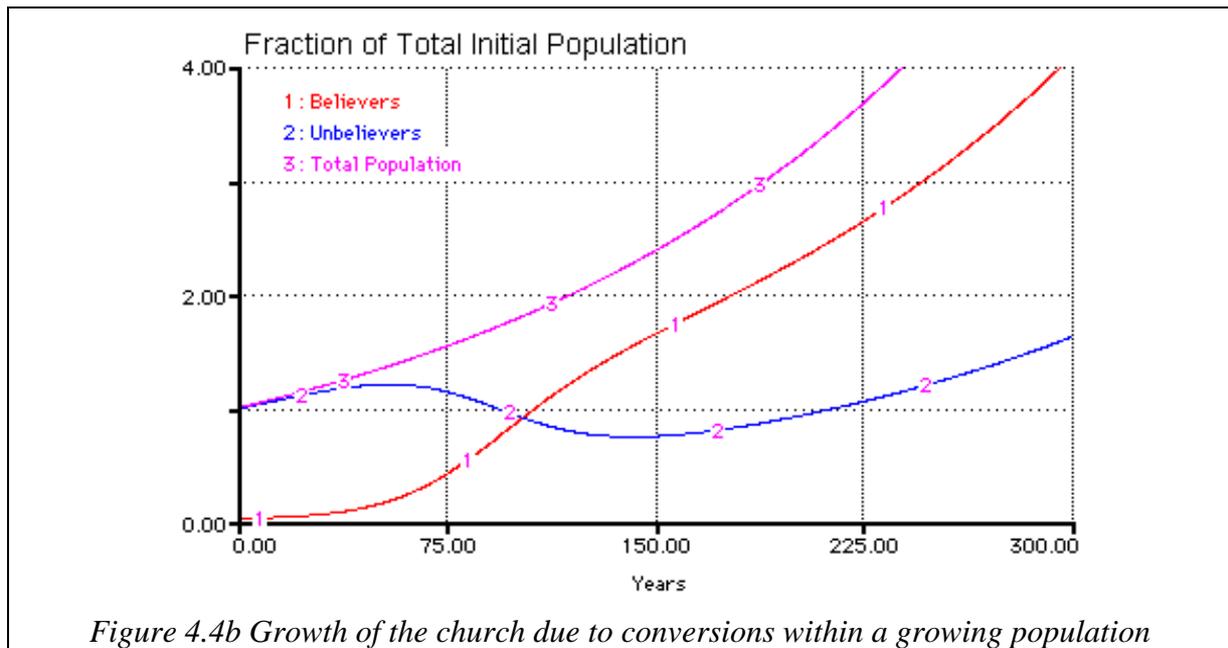


If the church is cannot hold its own children the conversion potential becomes crucial. Indeed it is possible that the conversion potential could be so low that the church eventually becomes extinct.

Consider the case where the church is already 50% of the population, and is only able to hold 50% of its children. The church then requires a conversion potential of 1.4% just to stand still. Anything lower and the church will decline. If it is too low it will effectively become extinct.

### 4.3.2 Birth Rate Larger than Death Rate

If the birth rate is allowed to be larger than the death rate then the population increases exponentially. Simulations show that although initially this favours the unbelieving majority there comes a point where the church makes enough inroads that its growth takes off and the number of unbelievers drops (figure 4.3d). Thus providing a church can maintain its conversion potential it will eventually see substantial growth within the growing populations.



With a higher conversion rate dominance is achieved sooner, but as with the case where the population doesn't grow, not all the population gets converted. There is always an unbelieving remnant.

#### 4.4 Conclusion

For the conversion model with unlimited enthusiasm, and equal birth and death rates:

- There are three parameters (discounting birth and death rates):
  - the potential number converted per believer per unit time;
  - the initial proportion of believers;
  - the fraction of children of believers who are kept in the church
- The number of believers rises (or declines) to a fixed, stable, value.
- That value is less than the total population unless the church is able to hold all its children in the faith. Thus the church losing some of its children will prevent all the population from being converted, and limit the growth of the church.

When the birth rate is bigger than the death rate:

- It can be a very long time before the growth of the church makes an impact on society, but it will eventually make an impact and dominate.

## 5 Reversion

### 5.1 Believers Leaving the Church

The next assumption that needs to be challenged is that believers remain believers all their lifetime. Clearly some do renounce the faith and return to the world. This may be for a number of reasons:

- Physical persecution. At its most extreme believer's lives could be in danger, as frequently happened in the early church. Christians would be asked to swear allegiance to Caesar and curse Christ or face death. Those who succumbed were no longer welcomed in the church. The fear of this test caused many others renounce the faith also.
- Psychological persecution. Some believers give up the faith due to pressure of unconverted family or peers. The pressure may take the form of teasing or being shunned from normal social activities. The believer gives up the church to have an easier life.
- Some of the believers were not truly converted in the first place and give up the faith once it loses its initial attraction.<sup>3</sup> The unbelieving world they left behind has proved more attractive to them.
- The believer may have fallen into sin and caused them to be ashamed to be part of the church anymore.
- The believer may have been hurt by others in the church and it is now too painful for them to be in the church with those who hurt them.

Thus assumption 4 is modified:

*Assumption 4.1*                      *Believers only remain believers for a fixed length of time*

This is an averaging effect. Many believers will remain believers all their lives, but some do not and return to the unbelieving world. This has the effect of reducing the *average* length of time a person remains a believer.

### 5.2 Permanently Hardened to Re-Conversion

#### 5.2.1 Construction

It now needs to be asked if this person who has reverted from the church can be converted again. If they are not open to conversion straight away then we have a new category of person, who can be described as hardened or resistant to conversion.

*Assumption 1.1*                      *There are three categories of people, unbelievers, believers and hardened unbelievers*

The model is now a three compartment one of unbelievers, believers and hardened (figure 5.2a). Further this introduces the rate at which people revert from the church. Reversion is

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<sup>3</sup> Both persecution, and the attraction of the unbelieving world, are part of the parable of the sower. The existence of those who look like believers but who are not truly converted is also implied in this parable as well as the parable of the wheat and the tares.

best described as a draining process where the numbers who revert are proportional to the population in the church.<sup>4</sup>

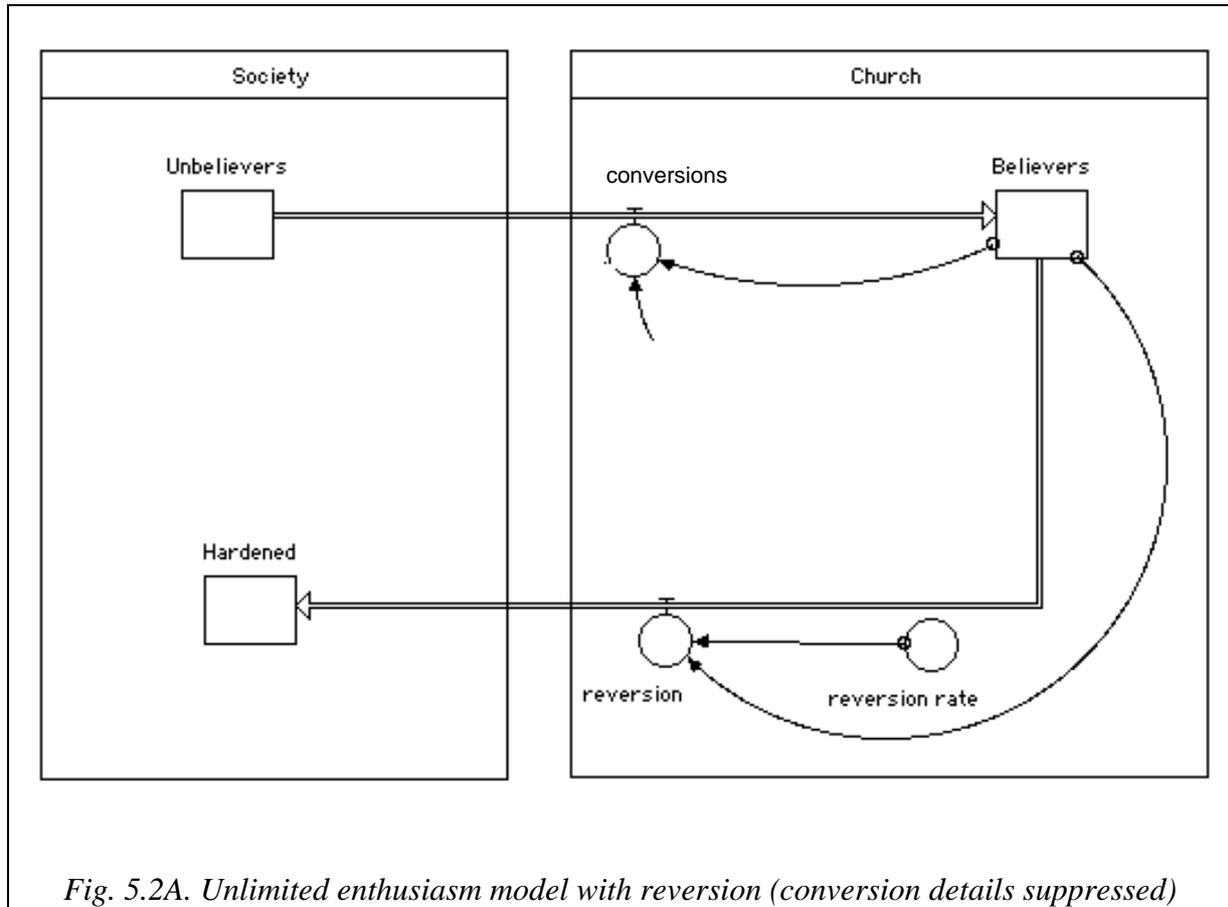


Fig. 5.2A. Unlimited enthusiasm model with reversion (conversion details suppressed)

As a first approximation it is assumed that those who revert are not open to re-conversion again. This may be true of certain sects that refuse to have ex-members back. It would also be true if state persecution removes some of the believers permanently from the influence of the church. Thus:

*Assumption 9.0*

*When believers leave the church they become permanently hardened to re-conversion back to the church.*

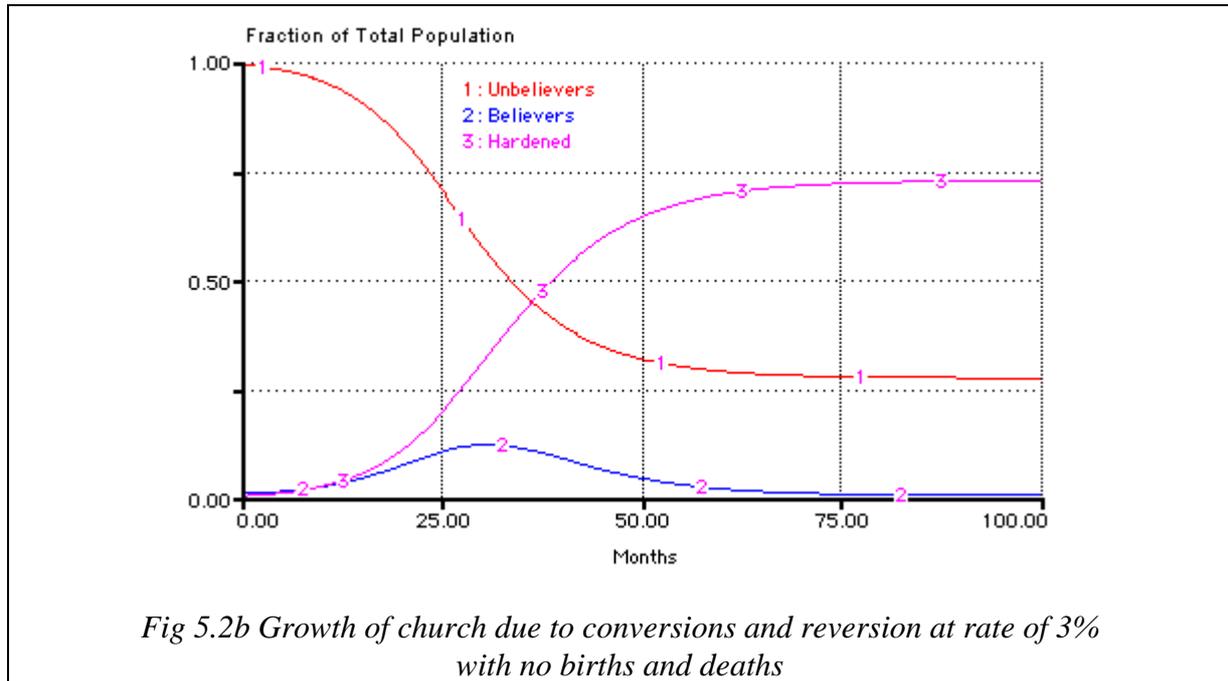
### 5.2.2 Short Term Behaviour

In the short term births and deaths can be ignored. This the model is effectively the epidemic model where the susceptibles are the unbelievers, the infected are the believers and the removed (or cured) are those hardened to conversion. Structurally this is the same as the model used in Hayward 1999 with the two main differences in that the middle category, the infected, constitutes the whole church and the final category, the removed, are people outside the church.

In the epidemic model the infected category always dies away leaving only the susceptible and the removed. Thus the church dies away after an initial growth, leaving some, but not all

<sup>4</sup> Poisson process

of society hardened to conversion (figure 5.2b). Clearly this is an unrealistic result as a time-scale has been chosen so that births and deaths can be ignored.



*Fig 5.2b Growth of church due to conversions and reversion at rate of 3% with no births and deaths*

However it is good model of short term fashion or behavioural patterns where there is no time to pass such things onto children, or where children would have no desire to copy adults, or where they have been deliberately excluded from exposure to the behaviour. Many sects and cults follow this pattern and die out for a failure to stem its losses. Because they are so extreme it is usually impossible for those who have reverted to come back. Either they are not welcome, or they are so damaged by the experience they stay well away. Unless the belief can be passed to the children or those lost to the movement can be re-converted then the sect must become extinct<sup>5</sup>.

### 5.2.3 Historical Considerations

The church itself has had many such short-term behavioural patterns. One such example was an early charismatic movement under the leadership of a Presbyterian minister Edward Irving in the 1820's (Dallimore 1983). For a brief period a number of people exhibited charismatic behaviour. In this application the “unbelievers” are church people and the “believers” are those with the behaviour. The movement eventually burned out, not affecting the wider church or the later Pentecostalism. A church of such people did remain, but only a shadow of its former self and now separated from mainstream church life, These are the equivalent of the hardened<sup>6</sup>.

<sup>5</sup> Many adventist sect follow this pattern. When the promised end of the world doesn't come disillusionment causes reversion and such people are rarely open to being re-converted into the sect again. Such movements only survive if their prophecies are re-interpreted as has happened with the Jehovah's Witnesses and Seventh Day Adventists. Of course the revised sect is no longer the same as the original.

<sup>6</sup> The surviving church was called the Catholic Apostolic Church. It still survives with an estimated 10 members in 3 congregations. The movement appears to have had no direct influence on modern Pentecostalism or charismatic Christianity. Irving was “rediscovered” after these movements started (Dallimore, 1983, p174-5).

### 5.2.4 Long Term Behaviour

If births and deaths are included a more realistic picture for the growth of the church is achieved. If the person is permanently hardened and never open to being converted back into the church then the tendency is for the church to make part of society hardened to the faith. Figure 5.2c is typical. Like the cases without reversion the numbers in the church level out to a fixed value, often declining to it after an initial overshoot depending on its initial percentage in society. Thus reversion, like births and deaths, causes the church not to convert the whole of society even when new unbelievers open to conversion are being born. However the church does not die away as the pool of unbelievers is being replenished by births.

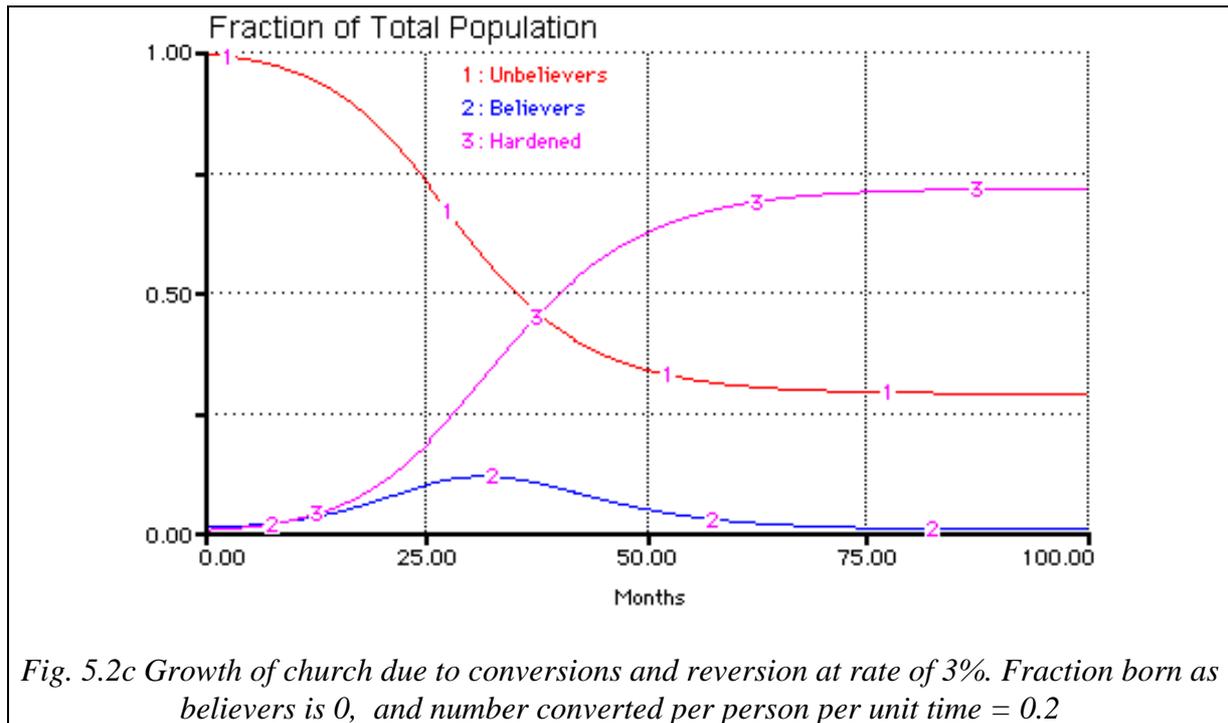


Fig. 5.2c Growth of church due to conversions and reversion at rate of 3%. Fraction born as believers is 0, and number converted per person per unit time = 0.2

Adjusting the conversion rate or the fraction who are born believers makes little difference to the final percentage in the church, or to the time-scale to reach stability. It has a bigger effect on the proportion of society hardened to conversion. The main effect on church numbers comes from adjusting the reversion rate. This rate of loss is therefore crucial for the long-term success of the church if there is no prospect or re-conversion.

The reversion rate in figure 5.2c was chosen at 3%. This means that the average length of stay in the church is around 30 years. This explains why it takes at least double that figure for the numbers to stabilise. Of course this average is made up of a small number of people who stay for much shorter periods than this and many who never leave the church at all.

### 5.3 Temporarily Hardened to Re-Conversion

The situation may not be as severe as described above. Some people having reverted from the faith only remain hardened for a length of time before again being open to re-conversion to the church. It usually takes time for hurts and fears to subside or for people to tackle restoring a broken relationship.

This temporary period of resistance to conversion may also be determined from the church's side. It is possible the church allows people a breathing space before making contact again or even deliberately shuns them for such a period.

Thus the hardened people undergo a “softening” enabling them to return to the pool of unbelievers open to conversion. Like reversion this is a draining process. Thus assumption 9 is modified:

*Assumption 9.1*                      *When believers leave the church they become temporarily hardened to re-conversion, becoming open to conversion again after an average length of time.*

Once they are open to conversion again is there a greater probability of being converted than someone who has never been part of the Christian church? Since they have more church contacts through their old church friends, and they have some understanding of the beliefs of the church it could be argued that they are more likely to be re-converted than a first-time convert. Against this has to be weighed the fact that some resistance, or hesitation at least, might still linger in their minds about re-joining a church. So some may be less likely to be re-converted. In the absence of any research to discover the true picture it is assumed that the rate of conversion of those who have fallen away is the same as unbelievers

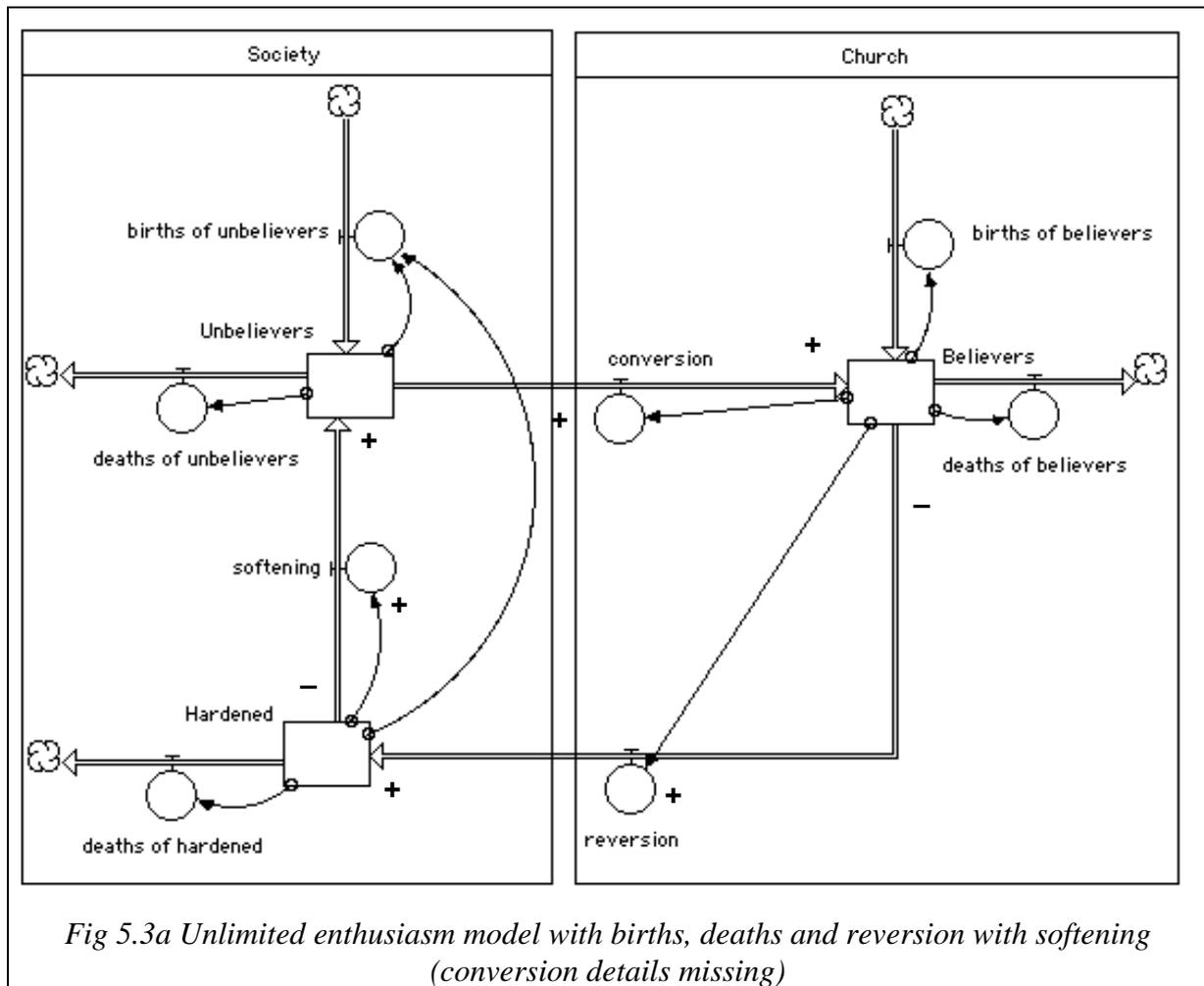
*Assumption 9.1a*                      *People who have left the church are just as likely to be converted as those who have never been converted before*

Thus the hardened people are fed back into the pool of unbelievers after their period of resistance to conversion is over.

It is also assumed that the children of hardened people are not born hardened but susceptible to conversion. Unless the families of hardened people have become isolated in some way this appears a reasonable assumption.

*Assumption 9.1b*                      *Children of hardened people are born as unbelievers.*

The complete conversion model with births deaths and reversion is given in figure 5.3a. Note that people are now physically re-cycled around the system through the three categories of people. This is a physical feedback loop.



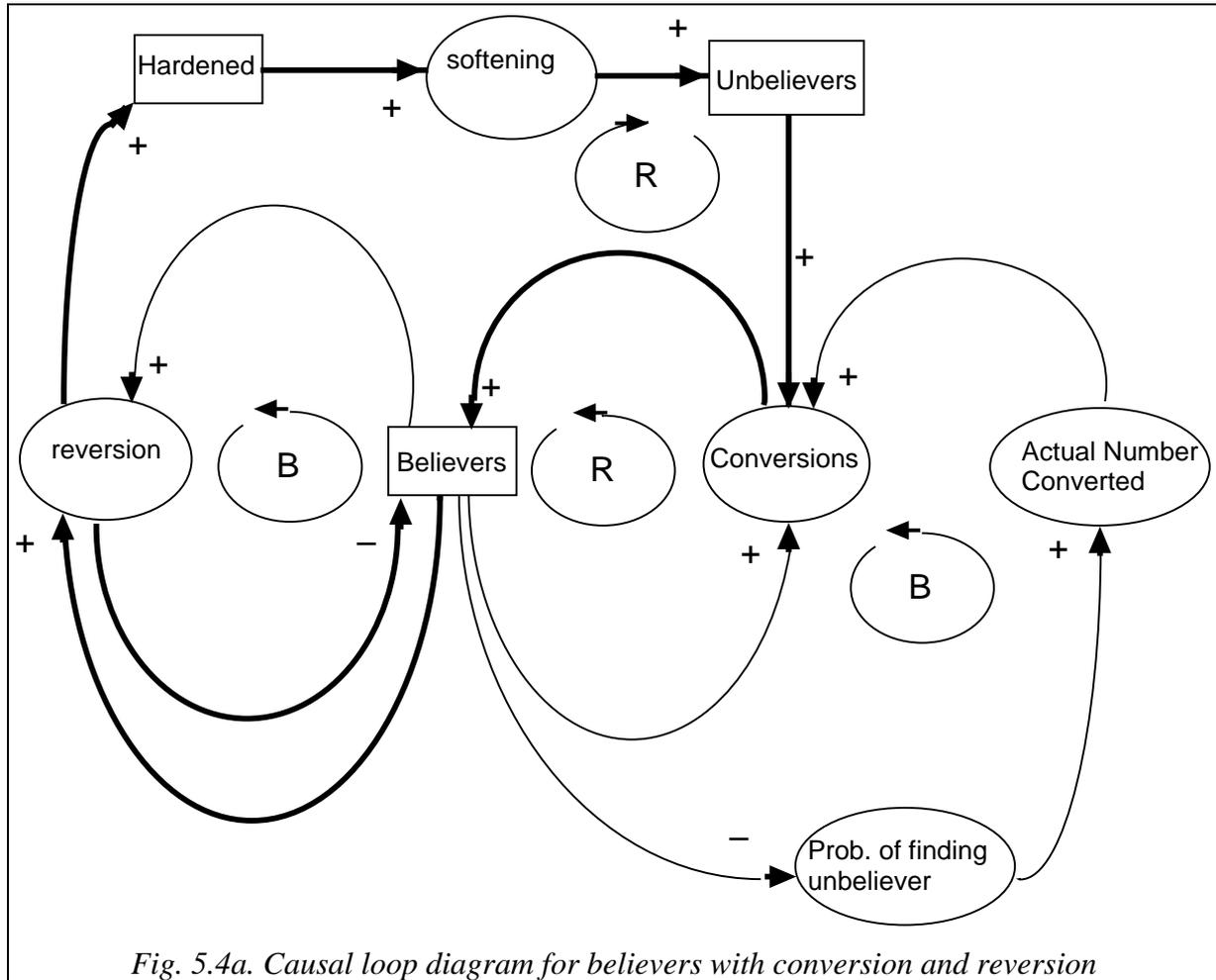
## 5.4 Analysis

### 5.4.1 General Considerations

To understand the effects of reversion with softening it is easier to ignore the births and deaths and examine how the causal loop diagram of 3.5a is modified. Reversion has introduced two new loops:

- The draining process because of reversion: Believers → reversion -. Believers. This is a balancing loop.
- The physical loop Believers → reversion → Hardened → softening → Unbelievers → conversion → Believers. This is a reinforcing loop.

This gives a very similar causal loop diagram (figure 5.4a) to the case with births and deaths. The fundamental reinforcing loop of believers through conversion is limited by the balancing one through the probability giving the S-shaped behaviour, as in the unlimited enthusiasm model of figure 3.5a. This time the balancing loop through reversion and back to believers lowers this limit by forcing reversion to balance conversions thus preventing the conversions getting to zero. As in the case of births and deaths this means the probability cannot be zero, thus some unbelievers must remain.



As in the case of births and deaths the possibility that unbelievers become zero is ruled out because of an extra reinforcing loop. This time the physical loop through all three categories of people re-cycles hardened people back into unbelievers through softening. If there were no softening this loop would not exist and believers would get brought to zero by the reversion loop. This was the case considered in section 5.2.

Thus the introduction of softening prevents the believers, i.e. the church, from declining to zero through reversion. Instead, just as with births and deaths, the stable limit reached by the church is less than the whole of society and some unbelievers and hardened people will be left. If births and deaths are included with reversion then the limiting effects are combined by merely adding the reductions from the two effects together<sup>7</sup>. Thus the church losing people, and it failing to keep its children could together significantly lower the level to which the church could rise.

<sup>7</sup> Both reversion and the birth death process are linear, i.e. effects are proportional to the population numbers. This enables the total reduction in the limit of believers to be the sum of the reductions of each of the two effects. Conversion is the only non-linear process in the dynamical system.

### 5.4.2 Implications of Reversion

To investigate further the effects of softening keep the same parameters as figure 5.2c and introducing a softening rate of 1%. There is now a considerable improvement in the final value of the church of 35% from 25% without softening (see figure 5.3c). Increasing the softening rate further sees substantial improvements in the churches numbers e.g. a softening rate of 6% sees the church up to 56%. Thus a church which is losing people back to the world can make considerable improvements in its long term prospects if it seriously considers how to win back those people, and makes the effort to keep the dialogue open.

Notice also that at the height of the growth the peak is more or less the same without softening (figure 5.2c) as with softening (figure 5.4b). Thus there is a temptation for the church in a successful period to ignore reversion because it has little short term effect on numbers. However it is the long term numbers that reversion effects. Notice in figure 5.4b, when there is no softening, the decline is much greater. The church is failing to win back the people who have left.

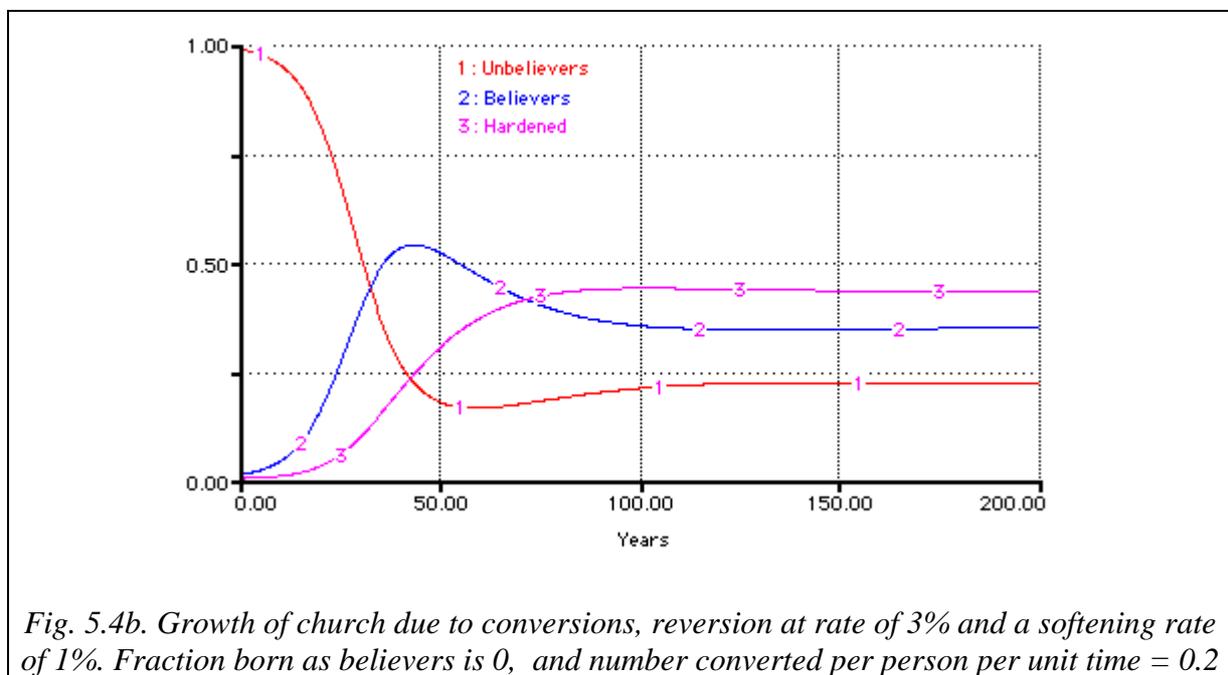


Fig. 5.4b. Growth of church due to conversions, reversion at rate of 3% and a softening rate of 1%. Fraction born as believers is 0, and number converted per person per unit time = 0.2

### 5.5 Church's Prospects With No Conversion

Clearly with no conversions taking place then growth of the church is not possible unless the church has a higher birth rate than surrounding society. Of course if there is no reversion and all the children remain in the church then the church will not decline. However what if there is reversion, could the church continue to maintain itself with a higher birth rate?

A typical situation is given in figure 5.5a. The church can indeed maintain itself, but only at the expense of unbelievers growing. Although it is possible for a church to grow in proportion to society by having a higher birth rate, once even a moderate amount of reversion takes place a substantially higher birth rate is needed to maintain the churches proportion in society. Trying to counterbalance reversion by the birth rate is highly unstable. Small changes in birth rate can make the difference between survival and decline. Believers are subject

purely to birth and death processes which either grow exponentially or declines to zero unless they just balance out. The conversion loop, which gave the stability by setting the target at or below the level of the total population is now missing.

The church losing a fraction of its children produces similar effects. Thus the effects of reversion, or losing children, can only be safely counteracted by conversion from the unbelieving pool. The prospects for any church that does not see conversions is very bleak unless it is so self contained that virtually no-one is lost and they consistently have larger than average families.

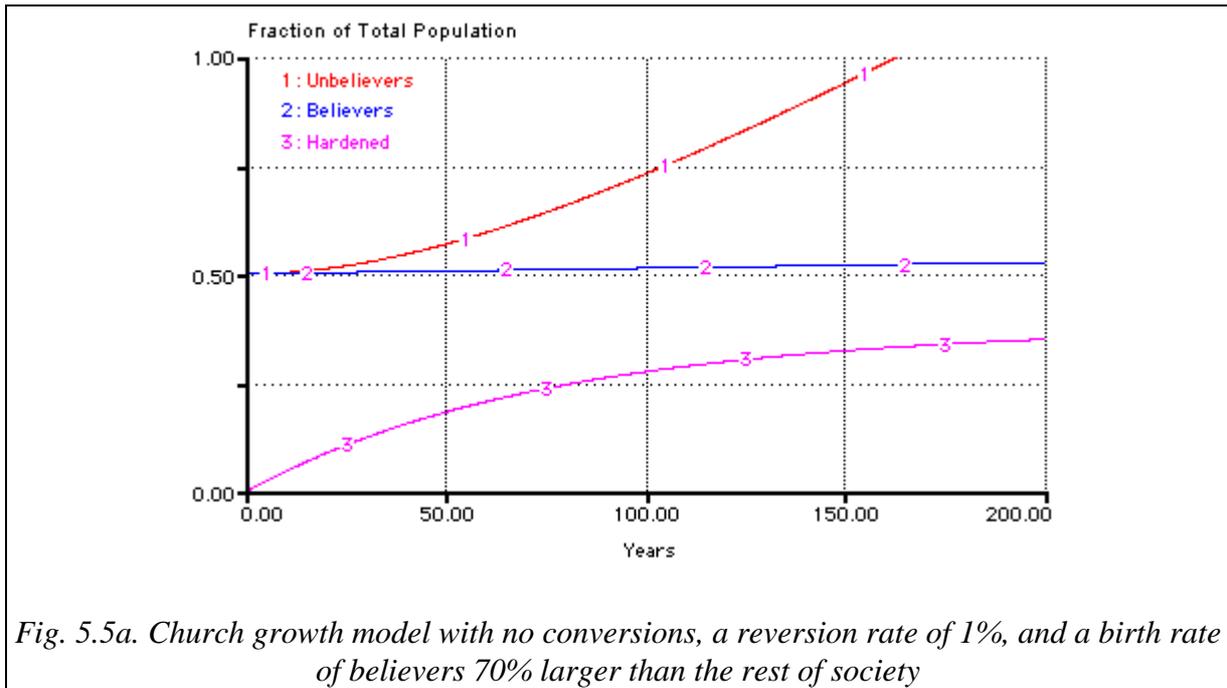


Fig. 5.5a. Church growth model with no conversions, a reversion rate of 1%, and a birth rate of believers 70% larger than the rest of society

### 5.6 Threshold of Extinction

Returning to the case where there the birth rates and death rates are equal, it is possible that if there may not be enough conversions to balance out the losses and allow the church to survive. This is not just a question of the church maintaining its current numbers, but it avoiding extinction altogether. It has already been noted that reversion and the loss of church children cause the church numbers to stabilise at a value less than the total population (e.g. figures 5.2c and 5.4b). If the losses are higher compared to the conversions, or the conversions are lower, then the church numbers may stabilise to zero. The church eventually comes extinct.

There is a threshold number of conversions under which extinction becomes inevitable. This is called the threshold of extinction. It is given by the formula:

$$\text{Potential number converted per believer per unit time (threshold)} = \text{reversion rate} + \text{fraction of children lost to the church} * \text{birth rate} \quad (\text{equation 5.6})$$

This comes from balancing out the gains and the losses to the believers. The number converted in one year must be equal to the number lost in the year due to reversion and the children of believers not continuing with church<sup>8</sup>.

Recent UK estimates give fraction of children lost to the church as one third (Brierley, 1991 p. 87) and the reversion rate as 0.016 (based on 1500 people leaving the church each week Brierley, 1999c, p.3). Using a birth/death rate of 0.014 this means the church must see a minimum of 2 converts for every one hundred Christians per year in order to avoid extinction. Over a lifetime this is about one and half new converts for every church member. It doesn't sound much, but this is to avoid extinction not to prevent decline. It must also be remembered that many Christians are never responsible for a single convert. This must be balanced by others bringing in more. This will be tackled in section 6 by having two categories of believers.

Currently the conversion rate in the UK is about the same as the reversion rate, 0.016. Thus it is significantly less than the required level to avoid extinction. The figures in Brierley (1999c, p.3) make this clear. The church is bringing in enough to balance out those who leave, but there are also deaths which are not balanced out by births as not all the children of believers are retained.

If the conversion rate were doubled to 4 converts in every hundred then the church would grow from its current 10% of the population to 30% in a hundred years. Halving the reversion rate to .008 has far less effect, the population only reaches 16% in a hundred years. Indeed reversion needs to be eliminated altogether to achieve a result as good as doubling the conversion rate. Thus near the threshold of extinction improving the rate of conversion has a larger impact than stemming the tide of losses.

Changes to the fraction of children retained by the church only makes small long term changes in the size of the church. However near the threshold of extinction this value can be crucial as well if the church is to survive. However if the church is growing with many conversions then the loss is less serious as many will get converted and return to the church later in life.

Interestingly the threshold of extinction does not depend on the softening rate. Thus the length of time people who revert from the church remain hardened does not affect the church's survival. It will only affect the level at which it survives and how long it takes to get to that level.

## 5.7 Conclusion

For the conversion model with unlimited enthusiasm, and equal birth and death rates and reversion:

- There are five parameters (discounting birth and death rates):
  - the potential number converted per believer per unit time;
  - the initial proportion of believers;
  - the fraction of children of believers who are kept in the church;

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<sup>8</sup> A mathematical analysis constructs this formula precisely

- the reversion rate;
- the softening rate.
- The number of believers rises (or declines) to a fixed, stable, value.
- That value is less than the total population. Thus reversion from the church, like the loss of children, will prevent all the population from being converted and limit the growth of the church.
- The church may rise to a high percentage before the effects of reversion cause it to stabilise at a lower value. Thus success in the church may initially mask the long term damage that reversion will cause.
- If the church has no conversions then its birth rate needs to be higher than that of surrounding society in order to balance the losses. However this is not stable and a small change in any one of the parameters can make the difference between growth and extinction.
- There is a threshold number of converts required to avoid extinction determined by the reversion rate and the fraction of children lost. If the potential number converted per person falls below this value the church eventually becomes extinct.
- Near the extinction threshold the long term levels of the church are more sensitive to the number converted per believer than it is to the reversion rate, or to the percentage born believers.
- The model is suitable if it is believed the bulk of the church are responsible for spreading the faith throughout their lifetime.

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